The edition of the text of the *Gaṇitasārakaumudī* is based on that of the Nahatas referred to above. The manuscript discovered by the Nahatas is the only known manuscript of the text, but, unfortunately, its whereabouts are no longer known. SaKHYa therefore only had the Nahatas' edition available when producing their book, and, as a result, what we have is a revised version of the Nahatas' edition. The Nahatas' text has been emended, when deemed necessary for mathematical or other reasons, but the original readings have been preserved in footnotes. Furthermore, words have been separated independently of phonetic changes, which makes it easier to find a word in the text.

The English translation presented in Part Three is literal and precise. The mathematical commentary in Part Four elaborates on the translation, contextualizing and explaining it. Finally, the appendices provide a concordance between the *Gaṇitasārakaumudī* and other works, a glossary-index to the text, and other useful tools.

Overall, the volume is a wonderful contribution to the field of the history of mathematics in India. The text is carefully edited, the translation precise, and the mathematical commentary solid and informative. Moreover, the introduction puts the material in the appropriate historical context. As a result, the volume will be of value both to the specialist, who will want to consult the original text, as well as to a more casual reader, looking to learn more about mathematics in India in the 14th century. It is hoped that SaKHYa will continue their good work on Indian mathematics and that their collaboration will produce more volumes like the present one.

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La construction tractionnelle des équations différentielles
By D. Tournès. Paris (Albert Blanchard). 2009. ISBN: 978-2-85367-247-4. viii, 406 pp. No price given.

Before the invention of computers and the development of numerical analysis, scientists and engineers conceived, designed and built various mechanical devices to speed up long, complex calculations. In recent years the history of these instruments has been the subject matter of different works, among which Dominique Tournès's *La construction tractionnelle des équations différentielles* certainly stands out. Tournès's book focuses on the integration of differential equations by tractional motion. This was a method of integration first used in the 1690s, forgotten in the second part of the eighteenth century, rediscovered in the nineteenth century and then forgotten again after 1950. This long story is interesting at least for two reasons. First, it shows the complexity of the interrelations between analysis, geometry, mechanics, and technology. Secondly, it clearly exemplifies that historical developments in mathematics often cannot be reduced to a mere accumulation of procedures, theorems, and findings.

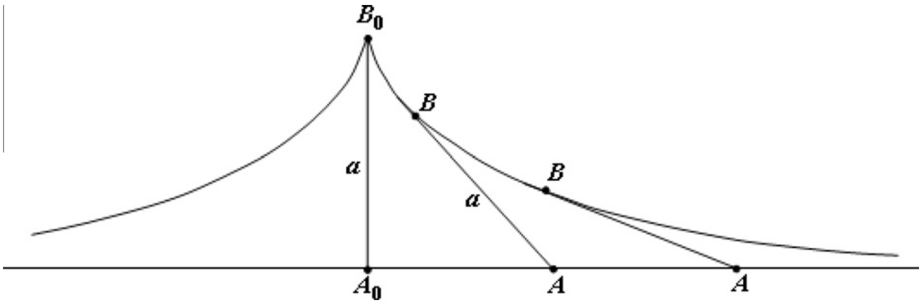

Fig. 1. Tractix.

Tournès's book contains three parts. First, chapters 1 to 3 discuss the evolution of the tractional construction of curves before 1752. The second part of the book (chapters 4 to 8) analyzes Vincenzo Riccati's (1707–1775) crucial memoir and its reception. Finally, chapters 9 to 12 are devoted to the rediscovery of the integration by tractional motion in the late nineteenth century and its developments in the early twentieth century.

The starting point was the investigation of a curve, the tractrix, whose characteristic feature is that the distance from any point B on the curve to the x-axis along the tangent at B is constant (see Fig. 1). At the end of the seventeenth century, mathematicians were interested in the tractrix because it can be used to draw the logarithmic curve by means of a continuous motion; indeed the differential equation of the tractrix is $\frac{dy}{dx} = -\frac{y}{\sqrt{a^2-y^2}}$ and therefore $x = a\ln\frac{a+\sqrt{a^2-y^2}}{y} - \sqrt{a^2-y^2}$.

Studies on tractional motion arose in the context where analysis was understood to be an instrument for solving geometrical problems. According to this conception, analysis investigated the relations between geometric quantities by means of analytical expressions; differential equations were thus understood to be in substance the analytical translation of a geometric problem. Therefore, analytical solutions to differential equations had to be translated into geometric terms. This operation was known as "the construction of the solution" and was performed by drawing and intersecting simple curves. Tournès observes (p. 224): "Une telle construction, de préférence réalisée par des mouvements continus simples, jouait un peu le rôle d'une preuve d'existence en analyse." The problem of the construction of equations was also connected with the question of the acceptability of curves in geometry. Indeed, in Descartes's geometry, non-algebraic curves could not be accepted, but, at the end of the seventeenth century, scholars more and more often encountered non-algebraic curves and felt that Descartes's demarcation did not fit the new development of mathematics and, in particular, the rise of the calculus. Thus, new methods for constructing non-algebraic curves were introduced. The construction by the tractrix and by curves that were generalizations of the tractrix was one of the new methods that helped legitimate transcendent curves (see Bos, 1988).

In 1693 Christiaan Huygens first published a study on the topic (in the form of a letter to H. Basnage de Beauval) in which he described the main characteristics of an instrument to draw the tractrix effectively. In the same year, Leibniz provided a first generalization of the idea of tractional motion and designed a mechanism that in theory could trace the integral curve of any given curves — a sort of universal integraph. In the decades that followed a number of mathematicians, including Jacob Bernoulli, the English John Perks, the Italians Giovanni Poleni (1683–1761) and Giambattista Suardi (1711–1767), described and sometimes built instruments by using tractional motion. In the history of the tractional construction of differential equations it is of great importance the integration of Riccati equations. Equations of the type $ax^n dx = dy + y^2 dx$ were introduced and studied by Jacopo Riccati (1676–1754) (it was d'Alembert who called the equation $\frac{dy}{dx} = Ay^2 + By + C$ *the* Riccati equation). During the eighteenth century many mathematicians studied them; in particular, in 1736 Euler provided the integration of Riccati equations by tractional motion. In Tournès's book the crucial character is Jacopo Riccati's son, Vincenzo Riccati (1707–1775), who in 1752 published a short memoir in Latin entitled *De usu motus tractorii in constructione aequationum differentialium*. Historiography usually attaches not much importance to Vincenzo's work. Henk Bos, for one, wrote that Vincenzo Riccati's contributions were essentially no more than simple generalizations and clarifications of Euler's procedures (p. 313). Tournès instead argues that while Euler's (and Clairaut's) works were the starting point for V. Riccati's *De usu motus tractorii*, yet his paper contains substantial novelties and presents a cogent synthesis of the works of the time on the resolution of differential equations by mechanical instruments. Tournès devotes the second part of the book to the analysis of Riccati's memoir. First, in chapter 4, he discusses the historic and scientific context of *De usu motus tractorii*, and then in chapters 5 to 8 analyses the content of the memoir and clarifies how Riccati derived successive generalizations of the concept of tractional motion that allow the integration of more and more extended classes of differential equations. By means of these generalizations, Riccati proved that any curve defined by a differential equation could be constructed by means of a tractional motion, and that there existed an infinity of different constructions. Of course, this result has to be understood taking into consideration the concepts of function and curve of the eighteenth century, deeply different from the modern ones (see Ferraro, 2004 and 2007). Tournès's French translation of the memoir is included as an appendix to the volume.

De usu motus tractorii constitutes the climax of the theory of the geometric construction of differential equations with the aid of simple continuous motions. Unfortunately, Riccati's investigations arrived too late, just when analysis was becoming an autonomous, self-founding mathematical discipline and was parting ways with geometry. After the 1750s the theory of the construction of curves fell into oblivion and "les tractoires, objets hybrides, ambigus, à la frontière de la géométrie et de la mécanique, font les frais du mouvement d'algébrisation de l'analyse" (p. 225). In the new context of Eulerian and Lagrangian analysis, Riccati's work lost its *theoretical* importance. For instance, when analysis was an instrument of geometry, the construction of a solution assured the existence of this solution, but in a sense it was an existence in the world of geometry. This was no longer acceptable after the 1750s, when geometrical concepts and even geometrical diagrams were banned from analysis. On the other hand, Riccati's memoir might have had a *practical* importance, since it contains a very general theoretical model to explain in a unified way the functioning of tractional integraphs; but this did not occur. It would be interesting to elucidate the reason for this lack of interest in the models for the construction of such instruments in the second half of eighteenth century. Tournès suggests that it was due to the improvement of numerical methods and to difficulties inherent in the use of integraphs, but he does not elaborate this interesting suggestion. Anyway, it is clear that after the 1750s the tradition of studies on tractional motion broke up and Riccati's work was forgotten. This leads me to an observation about the theoretical importance of Riccati's work. Tournès is right in criticizing Bos's opinion on V. Riccati's memoir (see above) and in claiming that Riccati's contributions were not mere generalizations and clarifications of Euler's procedures. On the other hand, it is right to say that Vincenzo Riccati did not grasp the new direction that late-

Enlightenment mathematics was taking. He was out of tune with the turn mathematics was making in his own days; his work became almost immediately old-fashioned and it is not surprising that mathematicians forgot it.

The history of tractional motion does not stop with Riccati. After a long period of eclipse, at the turn of the twentieth century mathematical practitioners were again interested in tractional motion and graphical integration, an interest that gave up only when the rise of computers relegated mechanical instruments of integration to museums. Tournès devotes the third part of his book to the rediscovery of the integration by tractional motion and its later developments. They were entirely independent of the seventeenth-century treatment, even if they were based upon the same theoretical principles and led to the same technical solutions. Tournès observes that while the greatest mathematicians dealt with tractional motion before 1750, after 1840 Industrial Revolution engineers were the ones who investigated tractional motion and used it to build instruments for solving differential equations. Tournès examines in detail the contributions of Gustave-Gaspard Coriolis (1792–1843), Bruno Abdank-Abakanowicz (1852–1900), Holger Prytz (1848–1930), Ljubomir Klertitj (1844–1910), Walter von Dyck (1856–1934), Louis-Frédéric-Gustave Jacob, Ernesto Pascal (1865–1940), and Emanuel Czuber (1851–1925), among others.

In conclusion, Tournès is to be congratulated for a well-written and organized contribution that will be certainly useful for those interested in the history of mathematics. It should have a place in any university library. Unfortunately it does not have an index so that information is often difficult to find. It is likely as well that by not being in English the book will not get as large an audience as it deserves.

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